Analysis in investment portfolios based on the Markowitz Model

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Abstract: Investing is a major trend in the financial world today. There are many types of investments and a wide range of combinations. Investing in the combinations that would provide the highest return is what everyone wants. To figure out the most efficient combination of the portfolios, we gather data of 10 companies from 5/11/2001 to 5/12/2021 and use the Markowitz Model to find out the best portfolio under 5 different constraints. The minimum variance has a return of 0.007979, and the maximum sharp has a return of 0.464124. The data and method we use may not be the newest, and there's still a need for improvement in the future for analysis.

1. Introduction

We cannot talk about the financial world without talking about investments [1-3]. Investing is involved in almost every decision. Famous investors like Benjamin Graham and Warren Buffett are not going to invest before analyzing returns. Some widely used investment methods are DCF analysis and leveraged buyouts [4-6].

In this article, we are going to use the Markowitz Model to better explore the applicability and effectiveness of the portfolio model.

We use the Markowitz Model and its 5 constraints to analyze the stock price of the past ten years from ten companies. We are going to introduce the ten companies, show the data in the remaining parts of the article, and explain and show the method and results of the best portfolio. The minimum variance has a return of 0.007979, and the maximum sharp has a return of 0.464124. The specific weights of each portfolio are shown in section 4.

The remaining of the article is organized as follows. Section 2 presents the background and the data we use; section 3 explains the method and equations of the Markowitz Model; section 4 presents the results of the analysis from section 3, and section 5 concludes the article.

2. DATA

The companies we did are NVDIA Corporation (NVDA), Cisco Systems, Inc. (CSCO), Intel Corporation (INTC), The Goldman Sachs Group, Inc. (GS), U.S. Bancorp (USB), The Toronto-Dominion Bank (TD CN), The Allstate Corporation (ALL), The Procter & Gamble Company (PG), Johnson & Johnson (JNJ), and Colgate-Palmolive Company (CL) from 5/11/2001 to 5/12/2021. The monthly stock prices for the past ten years are shown below. (Figure 1)

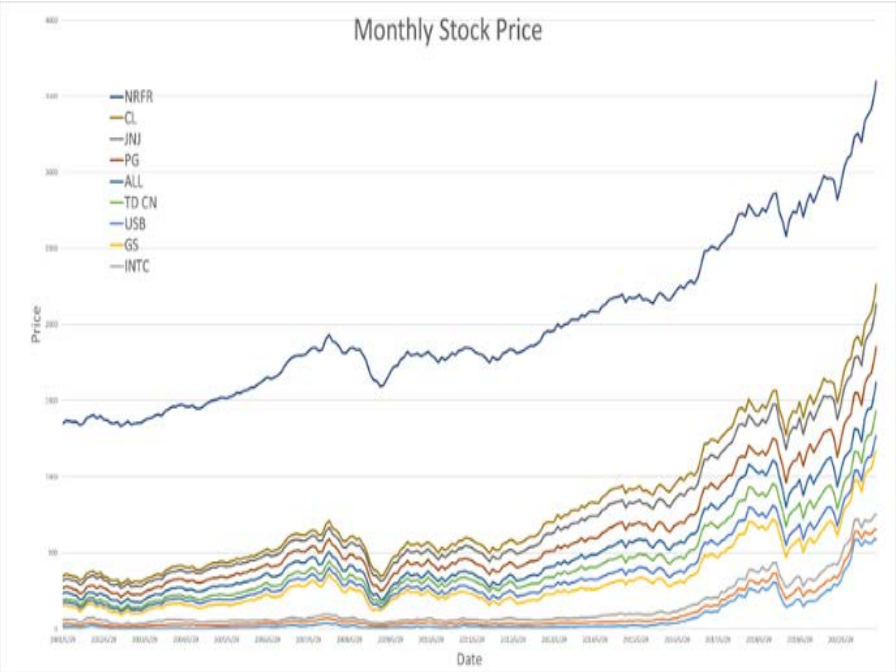

Figure 1 monthly stock price

NVIDIA: NVIDIA is very suitable for development in this field because many gamers are using the GPU designed by NVIDIA. The company has an enviable leading position in the d-GPU market for gamers. In addition, it recently launched the fastest gaming Graphics card ever. In October of this year, Nvidia released its next-generation cloud gaming platform GeForce RTX 3080 [7].

Cisco: In 1986, Cisco introduced the first multi-protocol router product. In 1990, Cisco went public. Since 1995, Chambers has served as Cisco's CEO and has created the Cisco Kingdom. In 2000, Cisco's development reached a climax and monopolized the world market for multi-protocol routers; Cisco once surpassed Microsoft in its market value to become the world's most valuable company, and Cisco's stock trading volume once surpassed the entire Shanghai stock market. In 2001, the Internet broke down, Cisco's performance fell, and its stock price fell by more than 80%. The company layoffs for the first time in its history. In 2003, Cisco developed rapidly in voip and other fields and developed an internal incentive entrepreneurial system, which enabled the company to continue its rapid development and return to its upward trajectory. In 2011, Cisco's performance was stagnant for a long time, and the outside world called for Chambers to resign. And by now, Huawei, which is a Chinese company had almost taken over its place [8-9].

Intel: Compared with Cisco and HP, Intel is such a focus turtle. In fact, Intel is often at the back end of the development of new technologies. Intel has a deep depth, and it was a huge industry, they take cost-efficiency seriously, so it's normal to turn around slowly and fail to catch up with fashion when others try their best to catch up with the trend, Intel stayed away and used the most cost-effective method to keep its opponents to death. Intel is the real overlord of Silicon Valley.

Goldman Sachs: an international leading investment bank and securities company that provides a wide range of investment, consulting, and financial services to the world. It has a large number of multi-industry clients, including private companies, financial companies, government agencies, and individuals. Founded in 1869, Goldman Sachs Group is one of the oldest and largest investment banks in the world. It is headquartered in New York, with branches in Tokyo, London, and Hong Kong, and has 41 offices in 23 countries. All its operations are built on a tightly integrated global basis, with outstanding experts providing services to customers. At the same time, it has a wealth of regional market knowledge and international operation capabilities [10].

U.S. Bancorp: U.S. Bancorp was founded in 1895 and is headquartered in Minneapolis. It is a financial service holding company. United Bank has assets of US$446 billion. Bank of America Corporation is the head office of Firstar Bank and Bank of America and is the fifth-largest commercial

bank in the United States. Provide complete financial brokerage services through its subsidiaries and ATM automatic teller machines, and provide consumers, enterprises, and institutions with banking, brokerage, insurance, investment, mortgage, trust, and payment services.

Toronto-Dominion Bank: Toronto-Dominion Bank is the second-largest bank in Canada by market value and deposits and the sixth-largest bank in North America. The group established the current Toronto-Dominion Bank Financial Group in 1955 by merging the former Bank of Toronto and the Toronto-Dominion Bank.

Allstate Insurance: It has high brand awareness in the United States and is one of the established insurance companies. Allstate Insurance Company provides a wide range of insurance products for American families, from common car insurance to family pet insurance, as well as life insurance that the U.S. Life Insurance Guide Network pays special attention to. Allstate has its characteristics.

The Procter & Gamble Company is founded in 1837 in Cincinnati, Ohio. It sells products in health and hygiene, personal care, beauty, home care, and family care. Before the sale of Pringles to Kellogg's, it also sold food and beverages. Procter & Gamble gained $83.1 billion in sales in 2014. The same year in August, it announced to streamline and drop off nearly 100 brands from its products portfolio to focus on the remaining 65 brands.

Johnson & Johnson: Founded in 1886, Johnson & Johnson is one of the world's most comprehensive medical and health companies with a wide range of businesses. Its business involves three major areas: medical equipment, pharmaceuticals, and consumer products. Headquartered in New Brunswick, New Jersey, USA, it has more than 260 operating companies in 60 countries and regions, with more than 130,000 employees worldwide. In 2020, global revenue will reach 82.6 billion U.S. dollars, and global R&D investment will reach 12.2 billion U.S. dollars.

Colgate-Palmolive is also a multinational products company. It is headquartered in Midtown Manhattan, New York City. It sells and produces products in household, house care, veterinary products, and personal care. In 2005, Colgate-Palmolive sold Fab, Dynamo, Arctic Power, ABC, Cold Power and Fresh Start, and Ajax brand to Phoenix Brands. In 2006, it announced to purchase Tom's Maine. Colgate-Palmolive now has organizations across almost 200 countries and is publicly listed in the United States, India, and Pakistan [11].

3. Method

We use the Markowitz model to find out the best portfolio under 5 different constraints.

By using the Markowitz model, we can find out the optimal portfolio of different stock combinations under different constraints, compare their results, and help investors' decision-making.

3.1. The Markowitz Model

The Markowitz model is a portfolio optimization model that helps choose the best portfolio among possible portfolios. By analyzing means and variances of the portfolio combinations, the Markowitz model finds the most return-to-risk efficient portfolio. The model includes a few assumptions: 1) The risk of a portfolio is based on the covariance of returns and 2) a single-period model of investment; 3) investors are supposed to be rational, averse to risk and 4) either minimize their risk for a given return or maximizes return for a higher risk.

To choose the most return-to-risk efficient portfolio, we need the efficient frontier—a hyperbola with the lowest risk, the tangency portfolio—a point of the optimal portfolio, the minimum variance frontier—a point where hyperbola changes from convex to concave, the minimum return frontier—a point that shows the maximum return in the given level of volatility, and the Capital Market Line—a tangency line of combinations of a risk-free asset.

$$Efficient\ Frontier: \begin{cases} r(\vec{w}) \to max_{\vec{w}} \\ subject\ to: \sigma(\vec{w}) = const \end{cases} \quad (1)$$

$$Minimum\ Variance\ Frontier: \begin{cases} \sigma(\vec{w}) \to min_{\vec{w}} \\ subject\ to: r(\vec{w}) = const \end{cases} \quad (2)$$

$$Minimum\ Return\ Frontier: \begin{cases} r(\vec{w}) \rightarrow min_{\vec{w}} \\ subject\ to: \sigma(\vec{w}) = const \end{cases} \quad (3)$$

$$Minimum\ Risk\ Portfolio: \{\sigma(\vec{w}) \rightarrow min_{\vec{w}} \quad (4)$$

$$Efficient\ Risky\ Portfolio: \left\{\frac{r(\vec{w})}{\sigma(\vec{w})} \rightarrow max_{\vec{w}} \quad (5)$$

3.2 Constraints

1)Regulation T by FINRA: $\sum_{i=1}^{11} |w_i| \leq 2$. To allow broker-dealers to let customers have positions, 50% of the regulation is funded by customers' account equity.

2) Arbitrary "box" constraint: $|w_i| \leq 1, for\ \forall_i$. The constraint simulates some arbitrary "box" constraints on weights provided by the client.

3) "Free" problem constraint: to see how the permissible portfolios and the efficient frontier look like without additional constraints.

4) Simulate open-ended mutual fund to avoid short positions: $|w_i| \geq 0, for\ \forall_i$. This constraint is designed to simulate the typical limitations in U.S. mutual fund industry and to avoid short positions.

5) Optimization Constraint: $w_1 = 0$. We use it to see the positive or negative effect in our portfolio affected by the inclusion of the broad index

4. Result analysis

Below is the Table of returns and the sharp ratio of the Markowitz Model (Table 1). The minimum variance has a return of 0.007979 and a standard deviation of 0.058897. The maximum sharp has a return of 0.464124 and a standard deviation of 0.449207. The weight of each portfolio is shown below.

Table 1 Return and Sharp Ratio of Markowitz Model

Markowitz	SPX	NVDA	INTC	GS	USB	TD CN	ALL	PG	JNJ	CL	Return	StDev	Sharpe
minVariance	-0.23561	0.009507	0.035495	0.025881	0.042062	0.012637	-0.03553	-0.12234	-0.12975	0.5	0.007979	0.058897	0.135467
maxSharp	-3.00464	0.024399	-0.22365	0.347671	0.360988	1.269344	0.215754	1.461627	1.167081	0.5	0.464124	0.449207	1.033207

5. Conclusion

We gathered the data of stock prices NVDIA Corporation (NVDA), Cisco Systems, Inc. (CSCO), Intel Corporation (INTC), The Goldman Sachs Group, Inc. (GS), U.S. Bancorp (USB), The Toronto-Dominion Bank (TD CN), The Allstate Corporation (ALL), The Procter & Gamble Company (PG), Johnson & Johnson (JNJ), and Colgate-Palmolive Company (CL) from 5/11/2001 to 5/12/2021. By using Markowitz Model under the five constraints mentioned above and analyzing the results, we are able to find out the best portfolio.

According to the analysis, the minimum variance has a return of 0.007979 and a standard deviation of 0.058897. The maximum sharp has a return of 0.464124 and a standard deviation of 0.449207.

The method we use is based on the Markowitz Model, which is not the newest method to find the most efficient portfolio. Therefore, the results would probably be different from the ones analyzed by other methods. In the future, and, after the pandemic, there may be different interpretations and methods to analyze the portfolios.

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